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# CONTACT INTERACTION OF CYLINDRICAL BODIES WITH ALLOWANCE FOR THE SURFACE ROUGHNESS FACTORS

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*The problem of the elastic contact interaction between a rough disk and a rough plane with a round cut-out is solved with allowance for the microgeometry of their surfaces. This makes it possible to clarify the effect of the main parameters of the problem on the stress state in the examined joint. A comparison of results from an analysis of the stress state in the contact region for various combinations of the elastic characteristics of interacting smooth bodies and results of well-known studies confirms the high effectiveness of the approach proposed.*

The advances in engineering pose new problems in the field of the serviceability of machines and their members. At the same time, the development of new computational models and the improvement of experimental methods require revision of the methods and assumptions used to determine the stress state of members. The solution of contact elasticity problems for bodies with circular boundaries and with characteristic linear dimensions of the contact region commensurable with the curvature radius of the contacting surface is of great significance for engineering practice. This solution is a theoretical basis of strength and rigidity calculations for machine components such as plain bearings, hinged joints, some types of gearing, and tension joints [1].

The equation of the contact problem for an elastic rough body was obtained for the first time by Shtaerman [2] assuming that in an elastic body, besides the displacements produced by the action of normal pressure and determined by solution of corresponding elastic problems, there are additional normal displacements due to purely local deformations depending on the microstructure of contacting surfaces. In a number of studies, it is assumed that the additional normal displacements due to deformation of microprojections of contacting bodies are proportional to the macrostress to some power. This assumption is based on equating averaged displacements and stresses within the base measured length of surface roughness. However, despite the well-elaborated apparatus for solving problems of these type taking into account the layer of elevated compliance [1, 2], some difficulties of a methodical nature have not been overcome. Thus, with allowance for the real microgeometry characteristics, the assumption of a power-law relation between stresses and displacements of the surface layer is valid for small base lengths, i.e., for a highly pure surface, and hence, with the validity of the hypothesis on topographic smoothness at the microlevel [3]. It should also be noted that this approach complicates the equation significantly and does not describe the effect of undulation. The accuracy of the coefficients in the integral equation obtained after the transformation $O(\varepsilon)$ is much lower than the accuracy $O(\varepsilon^2, 2\varepsilon/R)$ assumed in the formulation of the problem in [1] ($\varepsilon = R - r$, where $R$ and $r$ are the radii of cylindrical members), which contradicts the initial premises and greatly reduces the possibility of analyzing the effect of the parameter $\varepsilon$ on the distribution of contact stresses.

Thus, it is necessary to develop a unified approach to studying the contact interaction of cylindrical bodies with proper account of the geometry of interacting members, their relative position, surface micro-

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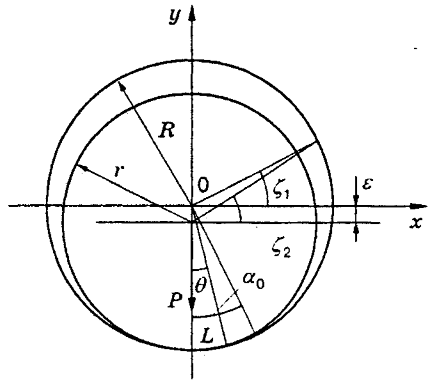


Fig. 1. Relative position of the contacting bodies.

geometry, and the possibility of obtaining an approximate solution with a small number of independent parameters describing the stress state in the contact region with sufficient accuracy for practice [1, 3, 4].

Following [5, 6], we assume that distortions of the shape of the interacting members are insignificant compared to the difference of their radii. We consider the problem of the interaction of an elastic isotropic layer of unit thickness having a round cut-out of radius $R$ with an elastic disk of radius $r$. We assume that $\varepsilon^2$ and $\varepsilon/R$ are small and a point force $P$ acting along the $y$ axis is applied at the center of the disk (Fig. 1). Friction is absent in the contact region $L$ ($-\alpha_0 \leqslant \theta \leqslant \alpha_0$, where $\alpha_0$ is the half-angle of contact).

We introduce the variables $x_1 = R\cos\zeta_1$, $y_1 = R\sin\zeta_1$, $x_2 = r\cos\zeta_2$, and $y_2 = r\sin\zeta_2 - \varepsilon$.

In the contact region $L$, we have $\zeta_1 = \zeta_2 = \zeta$ with the adopted accuracy (see [1] and Fig. 1). Since the displacements are small compared to the geometric dimensions of the bodies, the condition $(x_1+u_1)^2+(y_1+v_1)^2 = (x_2+u_2)^2+(y_2+v_2-\delta)^2$ is satisfied, where $u_m$ and $v_m$ are components of the displacement vector for the plane with a cylindrical hole ($m = 1$) and the disk ($m = 2$) and $\delta$ is the sag of the center of the disk. Ignoring small quantities of higher orders, we obtain $\varepsilon + u_1\cos\zeta + v_1\sin\zeta = u_2\cos\zeta + (v_2-\delta-\varepsilon)\sin\zeta$.

Let $u_m = u_m^* + u_m^{**}$ and $v_m = v_m^* + v_m^{**}$ ($u_m^*$, $v_m^*$, $u_m^{**}$, and $v_m^{**}$ are the displacements of the main material and the surface layer, respectively).

We assume that in the contact region, the elastic radial displacements due to the deformation of the microroughness are constant and proportional to the average contact stress $\sigma$ to some power $k$. This approach is widely used in studies of plane joints but the accumulated experimental material indicates that it can also be used for cylindrical members having similar radii [5, 6]. After elementary transformations taking into account the adopted assumptions, we obtain

$$\varepsilon - v - 2\frac{\partial u_1^*}{\partial\zeta}\sin\zeta + 2\frac{\partial v_1^*}{\partial\zeta}\cos\zeta + \frac{\partial^2 u_1^*}{\partial\zeta^2}\cos\zeta + \frac{\partial^2 v_1^*}{\partial\zeta^2}\sin\zeta$$

$$= -2\frac{\partial u_2^*}{\partial\zeta}\sin\zeta + 2\frac{\partial v_2^*}{\partial\zeta}\cos\zeta + \frac{\partial^2 u_2^*}{\partial\zeta^2}\cos\zeta + \frac{\partial^2 v_2^*}{\partial\zeta^2}\sin\zeta,$$

where $v$ is the approach of the interacting surfaces due to the deformation of microirregularities in the contact region.

In addition, for the main material, we have [7]

$$\frac{1}{R_m}\Big(\frac{\partial v_{\zeta m}^*}{\partial\zeta} + v_{rm}^*\Big) = \frac{1}{E_m}\,(G_{1m}\sigma_{\zeta m} - \nu_m G_{2m}\sigma_r) \qquad (m = 1,\ 2).$$

Here $R_1 = R$ and $R_2 = r$, $\nu_m$ is Poisson's ratio, $E_m$ is Young's modulus, $\sigma_{\zeta m}$ and $\sigma_r$ are components of the normal stresses, $G_{1m} = (1-\nu_m^2)$ and $G_{2m} = 1+\nu_m$ for plane deformation, and $G_{1m} = G_{2m} = 1$ for plane stress. Then, for the contact arc, the following equality holds:

$$\varepsilon - v + \frac{R}{E_1}(G_{11}\sigma_{\zeta 1} - \nu_1 G_{21}\sigma_r) + \frac{\partial}{\partial\zeta}\Big(\frac{\partial u_1^*}{\partial\zeta}\cos\zeta + \frac{\partial v_1^*}{\partial\zeta}\sin\zeta\Big)$$

$$= \frac{r}{E_2}(G_{12}\sigma_{\zeta 2} - \nu_2 G_{22}\sigma_r) + \frac{\partial}{\partial\zeta}\Big(\frac{\partial u_2^*}{\partial\zeta}\cos\zeta + \frac{\partial v_2^*}{\partial\zeta}\sin\zeta\Big). \tag{1}$$

We use the well-known relations [8]

$$\sigma_{\zeta m} + \sigma_r = 2[\Phi_m(w) + \overline{\Phi_m(w)}], \qquad \sigma_{\zeta m} - \sigma_r + 2i\tau_{r\zeta m} = 2exp(2i\zeta)[\bar{w}\Phi'(w) + \Psi(w)],$$
$$2\mu_m(u_m^* + iv_m^*) = æ_m\varphi_m(w) - w\overline{\Phi_m(w)} - \overline{\psi_m(w)}, \tag{2}$$

where $w = z$ ($m = 1$) for the plane and $w = s$ ($m = 2$) for the disk, $\mu_m$ is the Lamé coefficient, $i = \sqrt{-1}$, $æ_m = 3 - 4\nu_m$ for plane deformation and $æ_m = (3 - \nu_m)/(1 + \nu_m)$ for plane stress, $\varphi_m'(w) = \Phi_m(w)$ and $\psi_m'(w) = \Psi_m(w)$, where $m = 1$ and 2.

From (1) and (2) it follows that

$$\varepsilon - v + \frac{R}{E_1}(2G_{11}[\Phi_1(t) + \overline{\Phi_1(t)}] - (G_{11} + \nu_1 G_{21})\sigma_r) + R\frac{\partial}{\partial\zeta}\Big(\frac{æ_1 + 1}{4\mu_1}i[\Phi_1(t) - \overline{\Phi_1(t)}]\Big)$$

$$= \frac{r}{E_2}(2G_{12}[\Phi_2(h) + \overline{\Phi_2(h)}] - (G_{12} + \nu_2 G_{22})\sigma_r) + r\frac{\partial}{\partial\zeta}\Big(\frac{æ_2 + 1}{4\mu_2}i[\Phi_2 \tag{3}$$

$$1/h = R/(rt).$$

Taking into account the loading scheme (see Fig. 1), the absence of friction in the contact region, and the results of [1], we have

$$\Phi_1(z) = \frac{æ_1}{2\pi(1 + æ_1)}\frac{iP}{z} - \frac{1}{2\pi i}\int_L \frac{\sigma_r(\tau)\,d\tau}{\tau - z},$$
$$\Phi_2(s) = \frac{-iP}{2\pi(1 + æ_2)}\frac{1}{s} - \frac{iP}{2\pi(1 + æ_2)}\frac{s}{r^2} + \frac{1}{2\pi i}\int_L \frac{\sigma_r(\xi)\,d\xi}{\xi - s} - \frac{1}{4\pi i}\int_L \frac{\sigma_r(\xi)\,d\xi}{\xi}, \quad \frac{1}{\xi} = \frac{R}{r\tau}. \tag{4}$$

Thus, from (3) and (4) we obtain the following integrodifferential equation for the normal radial stresses in which all coefficients are determined with the specified accuracy:

$$\frac{t}{\pi i}\int_L \frac{\sigma_r'(\tau)\,d\tau}{\tau - t} = \gamma_1\sigma_r(t) - \frac{iP}{\pi}\gamma_2\Big(\frac{1}{t} - \frac{t}{R^2}\Big) - \gamma_3\frac{P}{\pi} - \gamma_4 b - \gamma_5(\varepsilon - v). \tag{5}$$

Here

$$\gamma_1 = \frac{(G_{12} - \nu_2 G_{22})E_1 Rr - (G_{11} - \nu_1 G_{21})E_2 R^2}{2(R^2 E_2 G_{11} + r^2 E_1 G_{12})}, \quad \gamma_2 = \frac{(1 + \nu_2)E_1 Rr + æ_1(1 + \nu_1)E_2 R^2}{4(R^2 E_2 G_{11} + r^2 E_1 G_{12})},$$

$$\gamma_3 = \frac{G_{12}\varepsilon R E_1}{2r(1 + æ_2)(R^2 E_2 G_{11} + r^2 E_1 G_{12})}, \quad \gamma_4 = \frac{G_{11}E_2}{R^2 E_2 G_{11} + r^2 E_1 G_{12}},$$

$$\gamma_5 = \frac{E_1 E_2 R}{2(R^2 E_2 G_{11} + r^2 E_1 G_{12})}, \quad b = -\frac{R^2}{2\pi i}\int_L \frac{\sigma_r}{\tau}\,d\tau, \quad v = C\Big(\frac{P}{2\alpha_0}\frac{1}{R}\Big)^k, \quad t = R\exp(i\zeta),$$

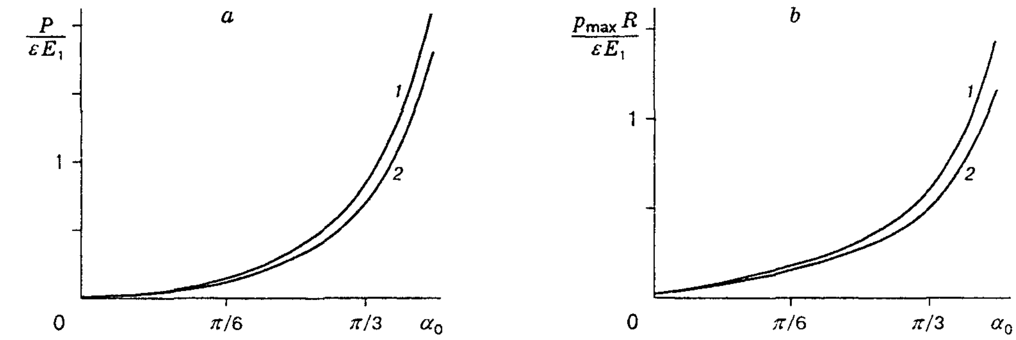


Fig. 2. Compressing force (a) and maximum contact pressure (b) versus half-angle of contact: curves 1 and 2 refer to smooth and rough surfaces, respectively ($R' = 3.12$ $\mu$m and $r' = 8.0$ $\mu$m).

and $C$ and $k$ are constants that depend on the roughness parameters $R'$ (height of smoothing) and $r'$ (calculated curvature radius of the protrusions) [6, 9, 10].

The approximate solution of the integral equation (5) is written as

$$\sigma_r(\theta) = -P\frac{\sqrt{2}}{R}\left[\gamma_2\frac{2}{\pi} + \frac{\gamma_1}{\alpha_0 - \cos\alpha_0\sin\alpha_0}\right]\sqrt{\cos\theta - \cos\alpha_0}\,\cos(\theta/2)$$

$$+2\left[P\left(\frac{\gamma_3}{\pi} - \frac{\gamma_1\cos\alpha_0}{R(\alpha_0 - \cos\alpha_0\sin\alpha_0)}\right) + \gamma_4 b + \gamma_5(\varepsilon - v)\right]\ln\left[\frac{\sqrt{1+\cos\theta} - \sqrt{\cos\theta - \cos\alpha_0}}{\sqrt{1+\cos\alpha_0}}\right]. \qquad (6)$$

The quantities $P$ and $b$ are evaluated from the equations

$$P = -2R\int_0^{\alpha_0}\sigma_r(\theta)\cos\theta\,d\theta, \qquad b = -\frac{R^2}{\pi}\int_0^{\alpha_0}\sigma_r(\theta)\,d\theta.$$

In solving the contact problem for smooth surfaces (plane deformation), it is established that when $\varepsilon \geqslant 0$ for various combinations of the elastic constants of the interacting bodies corresponding to the characteristics of isotropic materials used in engineering, the error of the approximate solution of Eq. (5) in the form (6) does not surpass 4% of the maximum contact pressure $p_{\max}$ [$p_{\max} = -\sigma_r(0)$], which is sufficient accuracy in solving practical problems. In addition, the dependences of the dimensionless parameters $P/(\varepsilon E_1)$ and $p_{\max}R/(\varepsilon E_1)$ ($\varepsilon/R < 0.05$) on the half-angle of contact (Fig. 2) agree well with the approximate solution of the problem obtained using four collocation points [1].

The solutions obtained for smooth bodies can be used as a zero approximation in studies of the effect of roughness on the distribution of normal radial stresses. Calculations show that in the case of interaction of rough bodies [9], the half-angle of contact increases somewhat compared to the half-angle for smooth members, and the highest contact stresses decrease (Fig. 2).

In design calculations of members, the studies performed make it possible to take into account not only the geometry of interacting bodies and their relative position but also the relief parameters specified by GOST 2789-73 "Surface roughnesses."

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